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HEAT FLUX DENSITY AS THE MAIN VECTOR IN THERMAL CONDUCTIVITY PROBLEMS

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ABSTRACT

The advantages of boundary conditions of the second kind in the problems of thermal conductivity and the method of presenting the results of analytical or research work in the form of heat flux density not as a product of the driving force of the process and resistance, but their ratio are shown. For the first time an analogue of the vector Terms for temperature fields – the vector of heat flux density was found. A brief overview of the development of thermometry in Ukraine and the transit calorimetry on its basis is presented. For closed-type calorimeters, recommendations are given for their design and fabrication using the Gauss-Ostrogradsky theorem, which relates the integral flow of a continuously deferenced vector field through a closed surface and the integral of the divergence of this field over the volume bounded by this surface. The Gauss's theorem for a isothermic-shell calorimeter (ISC) states that the total heat flux through its surface and the heat release or absorption capacity in the substance of the sample in the shell are the same, even if these fluxes are non-uniform over the surface and in the volume. The development of heat meters as small-sized, low-inertia sensors of heat flux density allowed the creation of thermal calorimeters-shells which common feature is the combination of the functions of the shell and calorimeter system. Operation of various types of ISC has confirmed their advantages over other calorimeters, namely: ISC shell has a small thermal resistance and inertia compared to the resistance and inertia of the sample, which allows to correctly study nonstationary processes; the calibration process is greatly simplified; the temperature differences are not measured at all; there is no need for differential measurements with a comparison sample etc.

1. Introduction

The modern development of food technology equipment is at a fairly high level. An important factor in improving existing and designing new food equipment is the study and study of the properties of food, in particular and thermophysical.

Thermophysical properties determine the nature and speed of the process of heating or cooling the product. These include specific heat, coefficients of thermal conductivity and thermal conductivity. Knowledge of thermophysical characteristics is required to calculate the amount of thermal energy required for cooling or freezing food during transportation, storage and processing.

The research objective is to show the advantages of boundary conditions of BC-2 of the second kind in thermal conductivity problems and to find the thermal analogue of vectors for energy flows of different types.

2. Materials and methods

Of the most common boundary conditions, let us mention the conditions of the first kind (BC-1), when the surface temperature of the solid body t_b is given, and of the second kind BC-2 – the given heat flux density through the surface q, the third kind BC-3 – respectively the ambient temperature t_a and the intensity of interaction between the body and the medium, and the fourth kind, when there is perfect thermal contact between the neighboring bodies. The most widespread are BC-3. In the research (Pekhovich and Zhidkikh, 1976) conditions of the fifth and sixth kind are proposed if there is a thin layer of solid or liquid substance with high heat capacity $c\rho$ with or thermal conductivity λ, and q or t are set already on the inner surface of this layer.

In solving problems of thermal conductivity with any boundary conditions, they are often reduced to BC-3

$$q = \alpha \cdot (t_b - t_a). \qquad (1)$$

This equation is still called the Newton's law, although BC-3 was introduced by Fourier. In addition α very often depends on $\Delta t = t_b - t_a$ (for example, under free convection conditions) or on q (during condensation on a solid surface). In addition, t_a in (1) is the temperature of the liquid (gas) outside the wall layer, and its thickness also depends on q and Δt. Therefore, Newton's relation (1) is an identical definition of $\alpha \equiv q/\Delta t$. The only convenience is that we can write $\alpha = 1/R_\alpha$, where R_α is thermal resistance of heat transfer and add R_α to other R in the heat transfer equation of Peclet.

E. F. Adiutori proposed to abandon the concept of "heat transfer coefficient" (Adiutori, 1977) and to investigate all heat transfer processes using the equation

$$q = f_1 \cdot f_2. \qquad (2)$$

where f_1 – system parameters; f_2 – thermal driving force.

For thermal conductivity, the thermal driving force is the temperature gradient, for heat transfer by radiation $T_{source}^4 - T_{receiver}^4$ (T is absolute temperatures of the source and receiver), and for convective heat transfer $t_b - t_a$.

The author (Adiutori, 1977) proposes to abandon also the numbers and similarity equations, the only exception is for free convection by introducing the Jenner number Je

$$Je = Nu\ Re = \frac{q\beta g l^4}{\nu\lambda\alpha}, \qquad (3)$$

that is, the dimensionless quantity q (the remaining quantities in (3) are the parameters of the system), which allows the experimental data to be processed in the form of equation (2).

For thermal conductivity, the functions f_1 and f_2 are unambiguous and equal to one

$$q = -\lambda \frac{dt}{dx}, \qquad (4)$$

is an equation of Fourier law (or the first Fourier law). This law also has limitations under conditions of unsteady intensive modes of heat transfer, its notation is complicated (Lykov, 1972):

$$q = -\lambda \frac{dt}{dx} - \tau_\rho \frac{dq}{d\tau}, \qquad (5)$$

where τ_p is the relaxation time – an analogous to the relaxation time of stresses that occur in the body under the action of deforming forces. It takes into account the rate of transfer of internal energy, which is not infinitely high, as is customary in the derivation of Fourier's law. For gases $\tau_p \sim 10^{-9}$ c for solids and liquids is even smaller (for metals $\tau_p \sim 10^{-11}$ c), so when considering the processes of heat power engineering and heat technology, the addend in (5) can always be neglected.

The gradient of any parameter in both space and time can play a greater role in a process than the parameter itself, not only in inanimate nature, but also in living organisms. It is written in the journal "Discoveries and Hypotheses" No. 9 of 2015 on page 22: "Living things in general tend to respond to the gradient rather than the modulus".

Despite the clear advantages of considering separate heat flux density and motive force instead of their ratio (heat transfer coefficient by radiation does not make physical sense at all), the new approach has not yet found a proper place in theory and practice, in monographs and textbooks.

In his monograph (Lykov, 1972) L. V. Lykov considers BC-2 only "when there is heating of bodies in high-temperature furnaces according to the Stefan-Boltzmann law", and for joint heat and mass transfer also under the same conditions of bringing the problem to BC-2. In the monograph (Carslow and Eger, 1964) all thermal conductivity problems of solids are solved from BC-3. In a very useful for researchers and designers work (Pekhovich and Zhidkikh, 1976) only 9 out of 78 problems reduced to computational graphs are devoted to BC-2. We hope that the development of thermometry will provide a proper impetus.

3. Results and discussions

Thermometry as a branch of thermophysics and metrology, designed to measure heat flux density q, W/m², originated in Ukraine 65 years ago. The first heat meters – the small-sized inertial heat flux density sensors were made in 1955 by spraying paired thermoelectrode materials on the pipe of a laboratory installation for the study of steam drying of Ukrainian earthy brown coal (Fedorov and Herashchenko, 1959). The signal of these so-called single heat meters was small, and they are still made for devices with high q. A sharp increase in sensitivity has been achieved in battery sensors of various types. The first author's certificate was obtained for a wafer-type heat meter (Fedorov and Herashchenko, 1963), where the principle of "parallel connection by heat flow and series connection by electrical signal" was implemented (Fig. 1). In the following years, these thermoelectric heat meters of the "auxiliary wall" type began to be mass-produced in Ukraine and other European countries, in the USA and Japan.

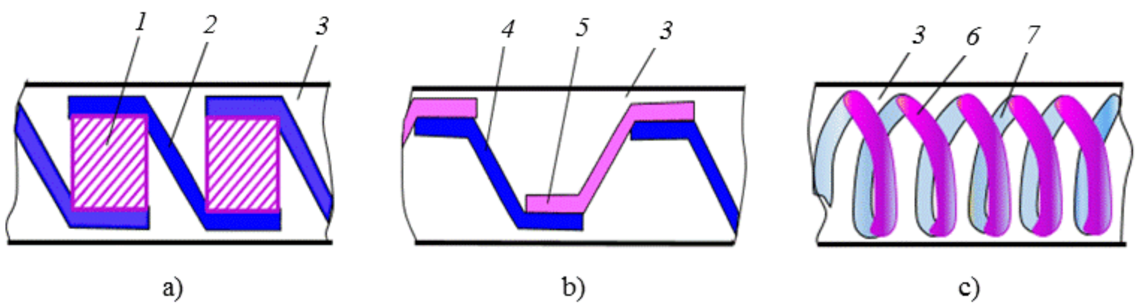

Figure 1. Sensitive elements of different calorimeters-shells
a) – wafer-type; b) – stepped; c) – spiral shells:
1 – constantan, 2 – copper, 3 – insulation, 4 – chromel, 5 – alumel, 6 – constantan covered with copper, 7 – pure constantan wire

On the basis of heat meters, several dozen types of derived devices have been created and implemented for measuring heat loss, determining thermal conductivity and heat capacity, radiation pyrometry, medical and biological research etc. Their use allowed reducing heat loss, insulation costs, determining the effective thermophysical characteristics (TPC) of new substances, correctly assessing the heat balance in thermal installations, effectively control and automating new technological processes (Herashchenko, 1971). The results of the study of processes in the food and refrigeration industry are summarized in (Fedorov, 1974), and in agriculture – in (Draganov *et al.*, 1993).

New information is not limited to heat transfer phenomena. Thus, the correlation between λ and the strength of fiberglass (Herashchenko, 1971), the hysteresis between λ and $c\rho$ of milk fat in (Fedorov *et al.*, 2020), is found. The results (Fedorov *et al.*, 2020) and many others were obtained using the flow thermometric calorimeters, the theory of thermal regimes, which is presented in (Fedoriv *et al.*, 1997).

A separate group among these calorimeters consists of closed flow isothermic-shell calorimeter (ISC). A fundamentally new feature of these ISCs is the combination of the functions of the shell and the calorimetric system. One of the first closed ISCs with a thermometric shell is shown in Fig. 2.

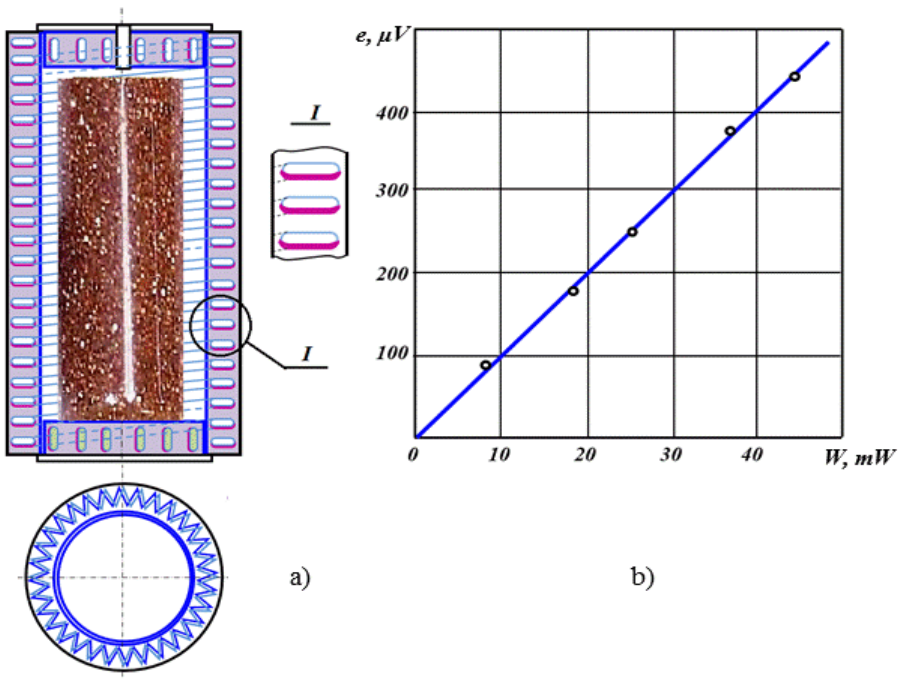

Figure 2. Scheme of microcalorimeter with thermometric shell (a) and its calibration characteristics (b)

The whole shell, including the cover, is made of series-connected heat meters, the electrical signal of such a battery is proportional to the average value of q passing through the shell to the sample in it, or vice versa. The issues of thermal interaction of the sample and the shell, the introduction of the necessary corrections to the output signal, as well as reference samples etc. led to the theoretical considerations. They were based on the Gauss-Ostrogradsky theorem. The Gauss-Ostrogradsky theorem connects the integral flux of a continuously differentiated vector field F through a closed surface S and the integral of the divergence of this field over a volume V bounded by this surface:

$$\iiint_V divF = \iint_S (F, n), \qquad (6)$$

where *n* are the coordinates.

Vector Condition is a vector of energy flux density of a physical field, which is transferred per unit time through a unit plane, which is perpendicular to the direction of energy flow at a given point (Ohorodnyk and Fedorov, 2020). Thus, the Vector Condition is a general concept of the quantitative characteristics of the transfer of different types of energy in any physical process. In particular, the vector of the flux density of the electromagnetic field is called the Poynting vector.

In thermophysics in general and in calorimetry separately, when considering thermal conductivity processes, such a vector should be the heat flux density q, W/m² – the amount of thermal (internal) energy transferred per unit time through a unit plane perpendicular to the direction of energy flow.

The use of the Gauss-Ostrogradsky theorem in this case is that it can be argued that the sums of heat fluxes through the surface of the shell calorimeter and inside, which is a sample in the shell, are the same, even if these fluxes are uneven on the surface and in volume.

The value of the surface integral in (6) should be determined by the signal of the shell – a calorimetric system consisting of a large number of elementary heat flux sensors connected in series.

Since the vector field $F = q$ is non-uniform, the components F must take q_x, q_y i q_z. By definition, the divergence of a field q is a scalar, which is a three-dimensional derivative of this field:

$$divq = \lim_{V\to 0}\frac{\int qdS}{V}. \qquad (7)$$

This value has a clear physical meaning – volumetric heat flux density, it has units of W/m³ and is denoted by q_V. We have the equation in Cartesian axials (8) for a closed shell-calorimetric system ISC:

$$\iiint\limits_V \left(\frac{\partial}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}\right)dxdydz = \iint\limits_S \left(q_x dydz + q_y dxdz + q_z dxdy\right). \qquad (8)$$

Both formally and physically, the result of summing both parts (8) is the integral heat flux *Q*, W, which passes through the shell and is equal to the thermal power, i.e. it is the amount of heat released or absorbed by the sample volume *V* and surface *S* per unit time.

The equality of the left and right parts (8) must be maintained during the design and operation of ISC, but this requirement is difficult to implement. The left part (8) requires equality of the sample volume and the internal capacity of the shell, which is possible only if the sample is a liquid, pasty or granular substance. A solid sample can be only if the energy enters it not by thermal conductivity, but in another way, for example, ionizing radiation. In this case, the signal of the thermometric shell must be corrected in the form of the ratio of the volumes of the sample and the shell. In other cases, the property of the air or other gas that fills the shell must be taken into account. The most accurate way to make this correction is calibration.

The right part of equation (8) can be a source of inequality of both parts due to the finite thickness of elementary heat meters and hence the entire heat meter shell. This part is a surface integral of the 2nd kind, i.e. the surface has two sides. Its orientation depends on the chosen direction of the vector q, but the thickness of this surface should ideally be zero. In all shell calorimeters, this thickness is an order of magnitude less than the characteristic size of the sample, but if the vector q is directed inside the shell, we must take the inner *S* and vice versa, which is outside then the outer *S* shell.

Summation and averaging of the shell signal is performed using modern electronic equipment.

If the design conditions do not allow making completely closed calorimeters, for example, when energy is supplied to the sample radially along a large length of pipe, the role of the end surfaces is played by the "protection zones" of the shell.

The prototype of the ISC is not a Mueller calorimeter, which measures the heat flux from the combustion chamber through a massive heat-conducting wall. Closer to the ISC are the Tian-Calvet calorimeters, in which the measurement of the temperature difference between the sample cell and the thermostated shell provides information about the heat flux.

The operation of different types of ISC has confirmed their advantages over other calorimeters:

a) the ISC shell has low thermal resistance and inertia compared to the resistance and inertia of the sample, which allows to conduct correct experiments during non-stationary processes;

b) difficulties in calibration are significantly reduced or disappear;

c) temperature differences are not measured at all;

d) there is no need for differential measurements with a comparison sample etc. ISCs are competitive in the global calorimeter market (Knauss H. *et al.*, 2006; Knauss H. *et al.*, 2009).

4. Conclusions

It is shown that during the theoretical or experimental solution of problems of thermal conductivity in a solid body it is necessary to pay more attention to boundary conditions of the second kind. For the first time on the basis of the Gauss-Ostrogradsky theorem for the processes of thermal conductivity an analogue of the Vector Condition is found – a surface heat flux density. The thickness of the thermometric calorimeter-shell can be reduced to millimeter, for example in modern oblique heat meters.

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