



## A NEW PARTICLE SWARM OPTIMIZATION MATHEURISTIC SOLUTION TO EMERGENCY FOOD DISTRIBUTION

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### ABSTRACT

The research of emergency food distribution and decision models mostly focus on deterministic models and exact algorithms. Some studies have been done on the multi-level distribution network and matheuristic algorithm. In this paper, random process theory is adopted to establish emergency food distribution and decision model for multi-level network. By analyzing the characteristics of the model, a modified discrete particle swarm optimization matheuristic algorithm (MBPSO) is proposed to solve the problem. In MBPSO, appropriate degradation mechanism and parallel global search structure are designed. Through an instance, MBPSO has a capability of global optimum search and fast convergence property for hybrid integer programming model with the multi-constrained and weighted single objective.

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### 1. Introduction

Emergency food distribution has been studied in extensive literature mainly focus on single-level network, deterministic models and planning algorithm, less research has been done on uncertainty in the emergency environment, multi-level distribution network and intelligent matheuristic algorithm.

R. Ji and Z. Xiao-lei formulate the problem through an integer programming model and a Lagrangian heuristic algorithm is developed to solve the problem (Ji and Xiao lei, 2014). Toyoglu et al., provide an ammunition distribution algorithm to a three-layer commodity-flow location routing formulation that distributes multiple products (Toyogluet et al., 2011). Naval warfare is studied in this literature (Gue, 2003), mainly concentrate on logistics center location, emergency food presets and emergency food distribution optimization problem. I. O. Pierskalla and W. P. propose

a three-index formulations for solving the problem of locating regional blood banks to serve hospitals (Pierskalla and P., 1979). in literature (Perl and Daskin, 1985) for designing the division's distribution system, in consideration of the system number, size, and locations of central depots.

Above these are some research achievements on distribution of emergency food. Most of them concentrate on the deterministic models and exact algorithm. Few of them focus on mataheuristic and matheuristic algorithm.

The authors propose also a matheuristic which aims at alternatively solving emergency food distribution design problem with estimated distribution amount, using exact methods, and determining the routing decisions and transportation, using heuristic procedures (Prodhon and Prins, 2014).

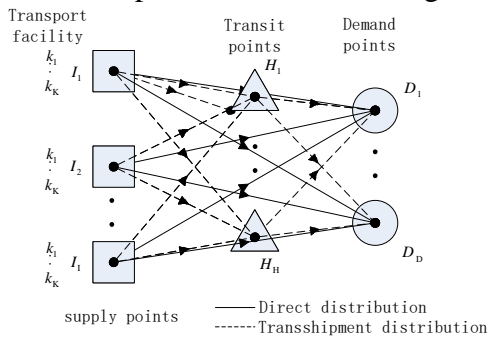
The main differences of our approach with other proposals existing in the literature

are that, in this paper, Poisson-Process is adopted to establish random risk of emergency food distribution (one of the three indexes). Meanwhile, an improved discrete particle swarm optimization (MBPSO) matheuristic algorithm is proposed to solve the problem.

The paper is organized as follows. In Section 2 we formulate the problems of emergency food distribution. and propose modified discrete particle swarm optimization algorithm (MBPSO). In Section 3, through an instance discussing different performance of three algorithms. Finally, Section 4 contains some conclusions and future research development.

## 2. Materials and methods

Emergency food distribution for Multi-level network is generally composed of three layers, supply points, transit points (depots) and demand points. As shown in Figure 1.



**Figure 1.** Three layers distribution network

In order to illustrate the model exactly, we first declare some symbol explanations.

$T$	Set of total time of distribution.	h
$x_{id}$	Set of amount of emergency supplies to be distributed to demand point $d$ at supply point $i$ , where $i \in I, d \in D$ .	t
$dis_{id}$	Set of distance between supply point $i$ and demand point $d$ , where $d \in D$ .	km
$dis_{ih}$	Set of distance between supply point $i$ and transit point $h$ , where $i \in I, h \in H$ .	km

$dis_{hd}$	Set of distance between transit point $h$ and demand point $d$ , where $h \in H, d \in D$ .	km
$v_k$	Set of velocity of transport facility $k$ , where $k \in K$ .	km/h
$t_{zz}$	Set of transshipment time.	h
$C$	Set of total cost of distribution.	
$c_{inv}^k$	Set of fixed cost of transport facility $k$ .	
$c_v^k$	Set of variable cost of transport facility $k$ , where $k \in K$ .	
$c_{zz}$	Set of the transshipment cost.	
$P$	Set of probability of being found by enemy in delivery paths.	
$S$	Set of random risk from enemy in delivery paths.	

### Greek Symbols

$\lambda_{id}$	Set of Poisson intensity between supply point $i$ and demand point $d$ , where $i \in I, d \in D$ .
$\lambda_{ih}$	Set of Poisson intensity between supply point $i$ and logistics transit point $h$ , where $i \in I, h \in H$ .
$\lambda_{hd}$	Set of Poisson intensity between transit point $h$ and demand point $d$ , where $h \in H, d \in D$ .
$\lambda_{i?d}$	1, if transport facility does not get through any transit point; 0 otherwise, where $i \in I, d \in D$ .
$\lambda_{i?k}$	1, if transport facility $k$ will be chosen, 0 otherwise, on condition that $\lambda_{i?d} = 1$ , where $i \in I, k \in K$ .
$\lambda_{i?h}$	1, if the transit point $h$ be chosen, 0 otherwise, where $i \in I, h \in H$ .
$\lambda_{ih?k}$	1, if transport facility $k$ will be chosen, 0 otherwise, on condition that $\lambda_{i?h} = 1$ , from supply point $i$ to logistics transit point $h$ , where $i \in I, h \in H, k \in K$ .
$\lambda_{hd?k}$	1, if transport facility $k$ will be chosen, 0 otherwise, on condition that $\lambda_{i?h} = 1$ , logistics transit point $h$ to demand point $d$ , where, $d \in D, h \in H, k \in K$ .

### Subscripts

$i$	supply point. $i \in I$
$h$	transit point. $h \in H$
$d$	demand point. $d \in D$
$k$	transport facility. $k \in K$
$v$	velocity

*inv*     invariable  
 ?        choose which one

Transport time and cost is divided into direct part and transshipment part. In every route in multi-level network, different transport facility will spend different time and cost, but only allowed to choose one of transport facilities in one route. If route pass through a transit point, it will produce transshipment time and transshipment cost, which associated to emergency food amount. The larger amount of emergency food will expend the more time and cost. The total time and total cost of emergency food distribution is shown in equation (1) and equation (2).

$$T = \begin{cases} \sum_{d \in D} \sum_{i \in I} \sum_{k \in K} \lambda_{i?d} \lambda_{i?k} \left( \frac{dis_{id}}{v_k} \right) & \lambda_{i?d} = 1 \\ \sum_{d \in D} \sum_{i \in I} \sum_{h \in H} (1 - \lambda_{i?d}) \lambda_{i?h} \left( \sum_{k \in K} \lambda_{ih?k} \left( \frac{dis_{ih}}{v_k} \right) + \dots \right. \\ \left. x_{id} t_{zz} + \sum_{k \in K} \lambda_{hd?k} \left( \frac{dis_{hd}}{v_k} \right) \right) & \lambda_{i?d} = 0 \end{cases} \quad (1)$$

$$C = \begin{cases} \sum_{d \in D} \sum_{i \in I} \sum_{k \in K} \lambda_{i?d} \lambda_{i?k} (c_{inv}^k + c_v^k x_{id} dis_{id}) & \lambda_{i?d} = 1 \\ \sum_{d \in D} \sum_{i \in I} \sum_{h \in H} (1 - \lambda_{i?d}) \lambda_{i?h} \left( \sum_{k \in K} \lambda_{ih?k} (c_{inv}^k + c_v^k x_{id} dis_{ih}) + \dots \right. \\ \left. x_{id} c_{zz} + \dots \right. \\ \left. \sum_{k \in K} \lambda_{hd?k} (c_{inv}^k + c_v^k x_{id} dis_{hd}) \right) & \lambda_{i?d} = 0 \end{cases} \quad (1)$$

$\{N(t), t \geq 0\}$  means a random number of destroy from uncontrollable factor in environment in multi-level network during period  $[0, t]$ . Assume that  $\{N(t), t \geq 0\}$  Obey strength  $\lambda$  for the Poisson Process, where the strength  $\lambda$  is mean to random number of destroy from uncertain environment to one of routes in unit time. In once destroy process, the probability of our emergency food being destroy is denoted by  $P$ , where  $0 < P < 1$ , and the event of environment

destroying emergency food within each time interval is mutual independent.

So we know that  $\{Y(t), t \geq 0\}$  belongs to Compound Poisson Process which obey the strength  $\lambda P$ . In order to investigate the random risk degree in transport process. We have to investigate some characteristic functions of this Poisson Process. In this paper, we choose the mean function as a target to evaluate the random risk degree of emergency food distribution.

Note that  $\{Y(t), t \geq 0\}$  obey strength  $\lambda P$  in Compound Poisson Process, so the mean function of  $\{Y(t), t \geq 0\}$  is shown in equation (3).

$$m_N(t) = E[Y(t)] = \gamma P t \quad (2)$$

Through equation (3) we can conclude that the expectation value of Compound Poisson Process is proportional to Poisson intensity, subsystems probability and duration time. Based on the above deduction, the random risk of emergency food distribution in multi-level network is shown below.

$$S = \begin{cases} \sum_{d \in D} \sum_{i \in I} \sum_{k \in K} \lambda_{i?d} \lambda_{i?k} \left( \frac{dis_{id}}{v_k} \right) \lambda_{i?d} P & \lambda_{i?d} = 1 \\ \sum_{d \in D} \sum_{i \in I} \sum_{h \in H} (1 - \lambda_{i?d}) \lambda_{i?h} \left( \sum_{k \in K} \lambda_{ih?k} \left( \frac{dis_{ih}}{v_k} \right) \lambda_{ih} P + \dots \right. \\ \left. \sum_{k \in K} \lambda_{hd?k} \left( \frac{dis_{hd}}{v_k} \right) \lambda_{hd} P \right) & \lambda_{i?d} = 0 \end{cases} \quad (4)$$

The objective function is shown in equation (5). It is consisted of time, cost and random risk these three indexes, on which we will put corresponding weight factor to cater to different needs in different scenario in section 3, we use uniformitarian process for objective function before evaluating its fitness. Some constraints are shown below.

$$\min Z = \alpha T + \beta C + \gamma S \quad (3)$$

$$S.T. \quad \sum_{h \in H} \lambda_{i?h} = 1 \quad \forall h \in H \quad \forall i \in I \quad (4)$$

$$\sum_{k \in K} \lambda_{id} \gamma_k = 1 \quad \forall i \in I \quad \forall d \in D \quad (7)$$

$$\sum_{k \in K} \lambda_{ih} \gamma_k = 1 \quad \forall i \in I \quad \forall h \in H \quad (8)$$

$$\sum_{k \in K} \lambda_{hd} \gamma_k = 1 \quad \forall h \in H \quad \forall d \in D \quad (9)$$

$$\sum_{i=1}^I x_{id} = x_d \quad \forall i \in I \quad \forall d \in D \quad (10)$$

$$\sum_{d=1}^D x_{id} \leq X_i \quad \forall i \in I \quad \forall d \in D \quad (11)$$

Constraints (6) describes that we can choose only one transit point. Constraints (7) demonstrates that we can choose only one transport facility, on condition that on condition that  $\lambda_{i \gamma d} = 1$ . Constraints (8) ensure that we can choose only one transportation from supply point  $i$  to transit point  $h$ , on condition that  $\lambda_{i \gamma d} = 0$ .

Constraints (9) ensure that we can choose only one transportation from supply point  $h$  to demand point  $d$ , on condition that  $\lambda_{i \gamma d} = 0$ . Constraints (10) guarantee that the emergency food amount of all supply points  $I$  to be distributed should be equal to the amount of demand point  $d$ . Constraints (11) exhibits that the total emergency food amount to be distributed in arbitrary supply point  $i$  should be less than their own inventory.

In recent years, many scholars proposed a variety of approach like heuristic algorithm, bionic intelligent algorithm, algorithm combined with the constraint condition etc, to investigate the problem (Mezura-Montes et al., 2010). But existing approach cannot solve our model. So, a new matheuristic algorithm will be proposed in this paper. In next section we will introduce a modified matheuristic algorithm to solve this problem.

**Improved Discrete Particle Swarm Optimization :** The particle swarm optimization algorithm (PSO) is proposed by Kennedy and Eberhart (Kennedy and

Eberhart, 1995). In PSO, a potential solution for a problem is considered as a bird, which is called a particle, flies through a D-dimensional space and adjusts its position according to its own experience and other particles'. In PSO, a particle is represented by its position vector  $p$  and its velocity vector  $v$ . In time step  $t$ , particle  $i$  calculates its new velocity then updates its position according to equation (12) and equation (13), respectively.

$$v_i^d = \omega v_i^d + c_1 r_1 (p_i^d - x_i^d) + c_2 r_2 (p_g^d - x_i^d) \quad (5)$$

$$x_i^d = x_i^d + v_i^d \quad (6)$$

where  $\omega$  is the inertial weight, and  $c_1$  and  $c_2$  are positive acceleration coefficients used to scale the contribution of self-cognitive and social-sharing components,  $v_i^d$  is the current speed value.  $v_i^d$  is the last speed value. respectively.  $P_i^d$  is the best position that particle  $i$  has been experienced in  $d$  dimensions.  $P_g^d$  is the best position found by all particles  $I$  in  $d$  dimensions.  $r_1$  and  $r_2$  are uniform random variables in range.

Standard PSO algorithm is suitable to continuously problem. In order to make the PSO algorithm more adaptive to solve discrete optimization problems, J. Kennedy and K. C. Eberhart, introduce a Binary-Particle Swarm Optimization (BPSO), which is more suitable for solving the problem of discrete.

BPSO algorithm inherits the velocity updating equation of the standard PSO algorithm. Firstly, utilize equation (12) to update the velocity value, then, SIGMOID function is used to convert velocity value into the probability of binary digit to get value 1. The process is shown below.

$$s(v_i^d) = \frac{1}{1 + \exp(-v_i^d)} \quad (7)$$

$$x_i^d = \begin{cases} 1 & \text{if } rand() \leq s(v_i^d) \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

Where  $rand()$  is uniform random variables in range [0,1]. It is necessary to set a maximum velocity  $v_{max}$  to limit the range of  $v_i^d$ , denoted by  $v_i^d \in [-v_{max}, v_{max}]$ .

The standard BPSO algorithm with fast convergence speed, but due to its following features, The particle population is easy to fall into local extremum. In order to overcome the deficiencies, we improved the standard BPSO optimization algorithm, making BPSO algorithm more efficiently and accurately search the global optimum solution.

**Solution Structure:** It is can be seen from Figure 2., The structure of the solution is divided into two parts ,the first part is linear programming part. Which represents the amount of emergency food to be distributed. The second part is heuristic part, which means combination of route and means of transport. Also, solving process is divided into two parts .Firstly, combination of transport means and routes is generated by MBPSO algorithm, then linear programming is used to find the best fitness of objective function.

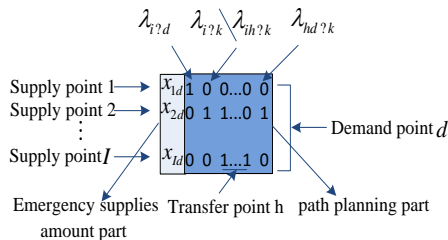


Figure 2. solution structure

**MBPSO Implement Steps :** Step1 Population initialization. In the problem definition domain, initializing population

position and velocity value randomly, and calculating its fitness.

Step 2 Stop judging. Stop and exit, if the algorithm meet stop condition. Otherwise, continue.

Step3 velocity and position update. equation (12), (14), (15) are used to update the velocity and position of populations and calculating new populations' fitness.

Step4 Parallel algorithm structure. At every predetermined sampling point, another parallel global search mechanism will be triggered in sampling period. Some separate population will be randomly initialized to get global optimum which denoted by Gbest1. In sampling period, if Gbest1 is better than Gbest, replace it.

Step5 Population degradation mechanism. At every predetermined sampling point. In roulette random way, with a certain probability substitute one of select particles' best solution it experienced (Pbest) for current global optimum (Gbest). Meanwhile, Storing Gbest. When sampling period is over, if there is no other better optimal value updated, then give the last stored Gbest back to the current optimal value.

Step 6 Evaluate fitness, Turn to Step 2.

### 3. Results and discussions

In order to verify the validity and practicability of the model and modified discrete particle swarm optimization algorithm (MBPSO), Constructing a three level emergency food distribution network, consists of 4 supply points, 3 transit point, 5 demand point and two kinds of transport facility.  $\alpha=0.5$ ,  $\beta=0.2$ ,  $\gamma=0.3$  is the weight of time, cost and random risk respectively. Transit costs and Transit time is that  $c_{zz} = 20\$/t$  and  $t_{zz} = 0.3h/t$ . Other parameters will be random initialized.

**Parameters Combination Experiment :** MBPSO algorithm performance is sensitive

to parameters setting of  $c_1$ ,  $c_2$  and  $w$ , which affects the convergence speed, accuracy and other properties. Therefore, combination value of  $c_1$ ,  $c_2$  and  $w$  is to be investigated firstly.

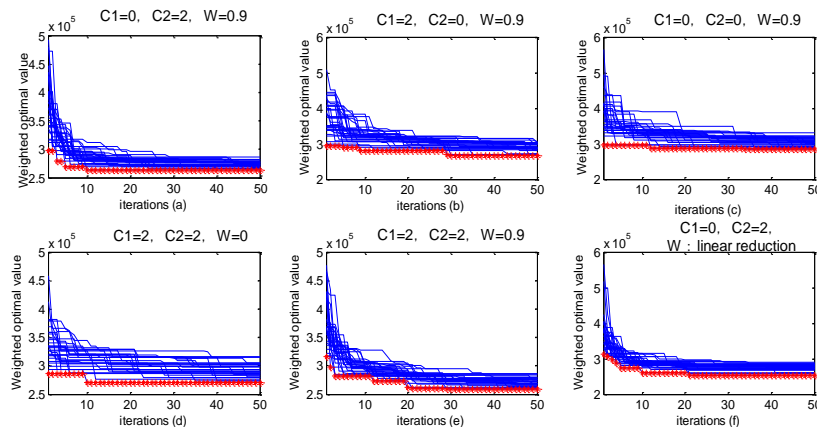
(1) parameters combination experiment

$w$  is the inertial weight, and  $c_1$  and  $c_2$  are positive acceleration coefficients used to scale the contribution of self-cognitive and social-sharing components. There have been lots of research on PSO algorithm parameters analysis, but without unified conclusion. For different question need to retest and reset the parameters.

According to the literature, parameter combinations test is designed for six groups, the result is shown in Figure 3 and Table.7

**Table 1.** MBPSO performance comparison with different parameter combinations

	C1	C2	W	Mean value
				*E+05
(a)	0	2	0.9	2.6422
(b)	2	0	0.9	2.6862
(c)	0	0	0.9	2.7542
(d)	2	2	0	2.7247
(e)	2	2	0.9	<b>2.5658</b>
(f)	2	2	0.9~0.1	<b>2.6365</b>



**Figure 3.** MBPSO particles convergence track under different parameter combinations

We can see that group (a) is a Social Model, because its parameter  $c_1=0$ , which presents the group (a) does not have the cognitive part. Similarly, group (b) is cognitive model. group (c) has neither society nor cognitive part, just has inertial weight. group (d) has both society and cognitive part, without inertial weight. **Eroare! Fără sursă de referință.** shows that the group(e) is the best on convergence accuracy. According to the literature, (Kennedy and Eberhart, 1997) In group(f), the inertial weight decreases linearly. It makes the algorithm running in the early stages can be carried out

large-scale global search, and later with a strong local search ability.

(2) Algorithms comparison

In order to verify the effectiveness of the algorithm comparison among MBPSO, BPSO and HCA was taken to be done In terms of calculation accuracy and convergence.

As shown in Figure 4 and

Table 2. The comparison among the HCA, BPSO and MBPSO in terms of calculation accuracy under different running time. As can be seen from sub-graph (a) in Figure 4, where MBPSO algorithm is

running under short time, and its performance is mediocre. The calculation accuracy trajectory of the three algorithms are close. The reason is because MBPSO algorithm does not take full advantage of degeneration mechanism and parallel global search function, it's structure is similar to BPSO algorithm, so its performance is not the best. whereas, with running a longer time, MBPSO algorithm degradation mechanisms and parallel global search function comes into play. It makes all

particles follow Gbest in local search, but also can rushed out of the local extremum restrictions to take global optimization. It is also can be seen from sub-graph (b,c,d), with running a longer time, the calculation accuracy trajectory of MBPSO algorithm starts better than the other two algorithms. Also we can conclude from variance image in Figure 4, with running a longer time, the stability of MBPSO is becoming better gradually.

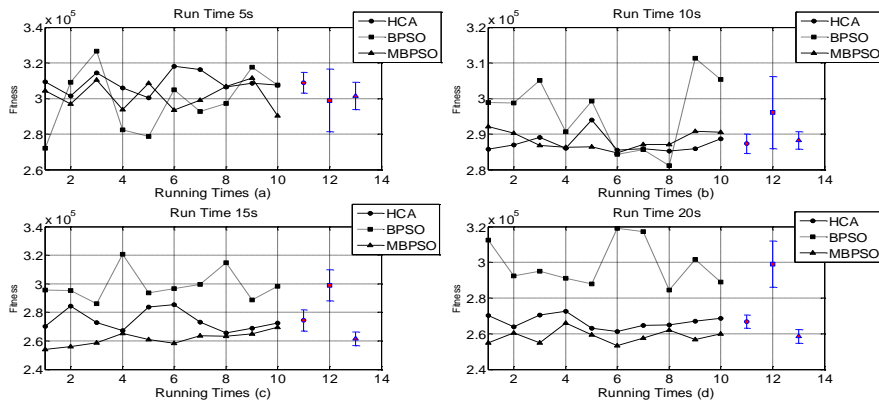


Figure 4. Algorithms' calculation accuracy comparison

Table 2. Mean value comparison of Algorithm calculation in 10 times run

run time	Algorithms	Mean value	run time	Algorithms	Mean value
		*E+05			*E+05
5s	HCA	3.0888	10s	HCA	<b>2.873</b>
	BPSO	<b>2.9887</b>		BPSO	2.9605
	MBPSO	3.0151		MBPSO	2.8822
15s	HCA	2.7439	20s	HCA	2.6671
	BPSO	2.9893		BPSO	2.99
	MBPSO	<b>2.6148</b>		MBPSO	<b>2.5854</b>

#### 4. Conclusions

On the premise of time and cost, Adopting Poisson-Process to establish emergency food distribution and decision model in multi-level network. By analyzing the characteristics of the model, On the basis of the standard discrete particle swarm optimization algorithm (BPSO), algorithm structure of appropriate degradation mechanism and parallel global search is designed.

In order to verify the effectiveness of the algorithm, contrasting (Mdfied Binary-Particle Swarm Optimizaion, MBPSO) with (Standard Binary-Particle Swarm Optimizaion, BPSO) and (Hill Climbing Algorithm, HCA), In terms of accuracy and convergence. The results show that (MBPSO) is of a global optimum and a fast convergence property for multiple constrained multi-objective integer programming model. But in the case of the running time is short, the

algorithm performance is almost the same with the other two algorithms.

In summary, there is a general and practical meaning for emergency food distribution and decision models in Multi-level network. Providing a new way for multi-constrained and multi-objective high dimensional optimization combination problem.

## 5. References

- Gue, K. R. (2003). A dynamic distribution model for combat logistics, *Computers & Operations Research*, 30(01), 367-381.
- Ji, R., Xiao-lei, Z. (2014). Logistic Supply Network Design in Battlefield Uncertain Environment with Enemy Attack Consideration, *Fire Control & Command Control*, 39 (6), 126-130.
- Kennedy, J., Eberhart, R. C. (1995). Particle swarm optimization, in Proc. IEEE Int. Conf. Neural Netw., 4, 1942-1948.
- Kennedy, J., Eberhart, R. C. (1997). A discrete binary version of the particle swarm algorithm, Paper presented at the Systems, Man, and Cybernetics, Computational Cybernetics and Simulation.
- Mezura-Montes, E., Coello-Coello, C. A. (2011). Constraint-handling in nature-inspired numerical optimization: Past, present and future, *Swarm and Evolutionary Computation*, 1(4), 173-194.
- Perl, J., Daskin, M. S. (1985). A Warehouse Location-Routing Problem, *Transportation Research Part B Methodological*, 19b(5), 381-396.
- Pierskalla, I. O., P., W. (1979). A Transportation Location-Allocation Model for Regional Blood Banking, *Iie Transactions*, 11(2), 86-95.
- Prodhon, C., Prins, C. (2014). A survey of recent research on location-routing problems, *European Journal of Operational Research*, 238(1), 1-17.
- Singh, H. K., Ray, T., Smith, W. (2010). C-PSA: Constrained Pareto simulated annealing for constrained multi-objective optimization, *Information Sciences*, 180(13), 2499-2513.
- Toyoglu, H., Karasan, O. E., Kara, B. Y. (2011). Distribution network design on the battlefield, *Naval Research Logistics*, 58(3), 188-209.

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